Assignment 6

Coverage: 15.8 in Text.

Exercises: 15.8. No 1, 3, 7, 9, 12, 14, 15, 16, 19, 20, 27.

Submit no. 7, 9, 12, 16, 19 by March 7. Yes , March 7.

Supplementary Problems

1. Find the volume of the ball $x^2 + y^2 + z^2 + w^2 \leq R^2$ in $\mathbb{R}^4$ by the formula

$$\text{vol} = \int_{-R}^{R} |B_w|\, dw \ ,$$

where $|B_w|$ is the volume of the cross section of the ball at height w. The answer is $\pi^2 R^4/2$.

2. Let D be a plane region in the plane unchanged under the map $(x, y) \mapsto (-x, -y)$. Show that

$$\iint_D f(x, y)\, dA(x, y) = 0 \ ,$$

when f is odd, that is, $f(-x, -y) = -f(x, y)$ in D. This problem has appeared in a previous exercise. Now you are asked to apply the change of variables formula in two dimension.

3. The rotation by an angle θ in anticlockwise direction is given by $(x, y) = (\cos\theta\ u - \sin\theta\ v, \sin\theta\ u + \cos\theta\ v)$. Verify that rotation leaves the area unchanged.

4. Consider the map $(u, v) \mapsto (x, y) = (u^2, v)$ which maps the square $R_1 = [-1, 1] \times [0, 1]$ onto $R_2 = [0, 1] \times [0, 1]$. Show that in general

$$\iint_{R_2} f(x, y) dA(x, y) \neq \iint_{R_1} f(u^2, v) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| dA(u, v) \ .$$

(Hint: It suffices to take $f(x, y) \equiv 1$.) Why?